M2

Interaction Photon-Matter

Exam, January 2010, duration 2 hours

1 Squeezed coherent states of the *em* field

Let be $\hat{D}(\alpha)$ and $\hat{S}(\xi)$ two operators defined as:

$$\hat{D}(\alpha) = e^{\alpha\hat{a}-\alpha^*\hat{a}^+}$$
$$\hat{S}(\xi) = e^{\left[\frac{1}{2}\xi^*\hat{a}^2-\frac{1}{2}\xi(\hat{a}^+)^2\right]}$$

where $\alpha = \alpha' e^{i\varphi}$ and $\xi = re^{i\theta}$ are two complex numbers.

A single-mode squeezed coherent state is defined as:

$$|\alpha,\xi>=\hat{D}(\alpha)\hat{S}(\xi)|0>$$

where $|0>$ is the ground state of the harmonic oscillator (or the vacuum state of the single *em* field mode in quantum optics context).

a) Verify that both $\hat{D}$ and $\hat{S}$ are unitary operators.

b) By using the Baker-Campbell-Hausdorff lemma find the expression of the operators $\hat{D}^+(\alpha)\hat{a}\hat{D}(\alpha)$ and $\hat{D}^+(\alpha)\hat{a}^+\hat{D}(\alpha)$.

c) From the above results find $\hat{D}^+(\alpha)\hat{x}\hat{D}(\alpha)$. Conclusion ?

d) Proof that $\hat{S}(\xi)\hat{a}\hat{S}^+(\xi) = \hat{a}\cosh(r) + \hat{a}^+e^{i\theta}\sinh(r)$ and find $\hat{S}(\xi)\hat{x}\hat{S}^+(\xi)$. For the particular case of $\theta = 0$ what do you conclude ?

e) In the following we consider that the *em* field is in a pure squeezed state $|0,\xi>$ with $\theta = 0$. Compute $<0,\xi|\hat{x}^2|0,\xi>$, $<0,\xi|\hat{x}|0,\xi>$, $<0,\xi|\hat{p}^2|0,\xi>$ and $<0,\xi|\hat{p}|0,\xi>$. Find $\Delta x\Delta p$. Conclusion ?

f) Compute $<0,\xi|\hat{\vec{E}}_\perp|0,\xi>$, $<0,\xi|\hat{\vec{E}}_\perp^2|0,\xi>$ for an arbitrary single-mode and find $(\Delta E_\perp)_{|0,\xi>}$. Conclusions ?

One gives:

$$e^x = \cosh(x) + \sinh(x)$$

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]] + \frac{1}{3!}[\hat{A},[\hat{A},[\hat{A},\hat{B}]]] + ...$$

$$\hat{\vec{E}}_\perp(\vec{r}) = i\sum_j \sqrt{\frac{\hbar\omega_j}{2\epsilon_0 V}}\left[\hat{a}_j\hat{\epsilon}_j e^{i\vec{k}_j\cdot\vec{r}} - \hat{a}_j^+\hat{\epsilon}_j^* e^{-i\vec{k}_j\cdot\vec{r}}\right]$$